Relativistic arbitrary-amplitude electrostatic solitons in a plasma

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(Received 9 March 1995)

A relativistic fluid model of a plasma in the case of a general polytropic process is considered. A "trajectory-boundary" method of analysis of electrostatic solitons is proposed as an alternative to the formalism of the Sagdeev pseudopotential, and is generalized to obtain an existence domain for compressive relativistic solitons in a two-component plasma.

PACS number(s): 52.35.Mw, 52.35.Sb

I. INTRODUCTION

The study of arbitrary-amplitude traveling structures (such as solitons, double layers, etc.) in plasmas has been a subject of considerable interest in recent years. The assumption of the arbitrariness of the structure amplitude makes the Korteweig–de Vries equation inapplicable, and one should use the more general fluid model. Starting from the work of [1], the nonrelativistic traveling solutions of the system of plasma fluid equations were usually analyzed in terms of the formalism of the Sagdeev pseudopotential. In the approximation of Boltzmann electrons and cold ions, it was shown in [2] that plasmas consisting of single ion and electron components do not admit nonrelativistic rarefactive solitons, although the question of their existence in a more general case remained open. Recently there appeared several publications where nonrelativistic traveling structures were examined in a multicomponent plasma [3,4]. A question of special interest has been the domain of existence of such solutions. Numerical investigations [5,6] have shown the existence of considerable restrictions on the range of parameters for such solutions to be possible. An alternative "trajectory-boundary" method was developed in [7], in which the authors succeeded in proving that plasmas consisting of two species do not admit rarefactive solitons. It was also shown that a two-component plasma with the same thermodynamic properties of the components cannot support double layers.

In this work the existence conditions for the traveling *relativistic* solitons are studied. The outline of the paper is as follows. In the next section, the relativistic fluid model of a plasma is introduced. In Sec. III, a partial integration of the model using a "traveling structure" ansatz is performed. This results in two constraints on the velocities configurations. The first one defines a curve and the second defines a region in velocities space. In Sec. IV, a necessary and sufficient condition for the existence of solitons in terms of the mutual geometrical locations of the constraints is formulated. As a by-product of this consideration, the nonexistence of rarefactive solitons is established. The existence domain of compressive relativistic solitons is established in Sec. V, and, as an example, it is found for an electron-positron plasma in Sec. VI. Finally Sec. VII is devoted to concluding remarks.

II. THE MODEL

The plasma is assumed to be infinite, homogeneous, unmagnetized, and neutral. Then, the system of plasma fluid equations is

$$\frac{\partial n_j}{\partial t}+\frac{\partial(n_j v_j)}{\partial x}=0, \tag{2.1}$$

$$n_j\left(\frac{\partial P_j}{\partial t}+v_j\frac{\partial P_j}{\partial x}\right)+\frac{T_j}{n_{0j}^{\gamma_j-1}}\frac{\partial n_j^{\gamma_j}}{\partial x}=-e_j n_j\frac{\partial \phi}{\partial x}, \tag{2.2}$$

$$\frac{\partial^2\phi}{\partial x^2}=-4\pi\sum_j e_j n_j. \tag{2.3}$$

Here n_j, m_j, e_j, v_j, and, γ_j are the density, mass, charge, velocity, and polytropic index of the species j, respectively, and P_j is relativictic momentum of species j

$$P_j=\frac{m_j v_j}{\sqrt{1-v_j^2/c^2}}\left(1+\frac{T_j}{m_j c^2}\right), \tag{2.4}$$

where c is the speed of light.

The boundary conditions are

$$\phi,\frac{\partial\phi}{\partial x},v_j\to 0;\quad n_j\to n_{j0}, \tag{2.5}$$

as $x\to-\infty$, where n_{0j} is the unperturbed density, which satisfies the neutrality condition

$$\sum_j e_j n_{0j}=0. \tag{2.6}$$

III. ARBITRARY AMPLITUDE TRAVELING STRUCTURES

Looking for a traveling structure propagating with a constant velocity u, it is advantageous to transform to a moving frame. With the new variable $\xi=x-ut$, Eq. (2.1) can then be integrated

$$v_j=u\left(1-\frac{n_{0j}}{n_j}\right).$$

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Introducing dimensionless parameters, $w_j = v_j/c,\ \nu = u/c,\ \tau_j = T_j/(m_j c^2)$, Eq. (2.2) can be written as

$$m_j c^2 \left[\frac{\gamma_j \tau_j (1-w_j^2)^{3/2} \left(\frac{\nu}{\nu - w_j}\right)^{\gamma_j - 1} - (1+\tau_j)(\nu - w_j)^2}{(1-w_j^2)^{3/2}(\nu - w_j)} \right] \frac{dw_j}{d\xi} = -e_j \frac{d\phi}{d\xi}. \tag{3.1}$$

Integrating it and taking into account the boundary conditions, one obtains

$$m_j c^2 \left[(1+\tau_j) \left(\frac{(1-\nu w_j)}{\sqrt{1-w_j^2}} - 1 \right) + \tau_j \ln\left(\frac{\nu}{\nu - w_j}\right) \right] = -e_j \phi \tag{3.2}$$

for $\gamma_j = 1$ (isothermal process), and

$$m_j c^2 \left\{ (1+\tau_j) \left(\frac{(1-\nu w_j)}{\sqrt{1-w_j^2}} - 1 \right) + \frac{\tau_j \gamma_j}{\gamma_j - 1} \left[\left(\frac{\nu}{\nu - w_j}\right)^{\gamma_j - 1} - 1 \right] \right\} = -e_j \phi \tag{3.3}$$

for $\gamma_j \neq 1$ (unisothermal process). Eliminating ϕ from Eqs. (3.2) and (3.3) results in

$$\begin{aligned} \mathcal{T}(\mathbf{w}) \stackrel{\text{def}}{=} \sum_j^{\gamma_j = 1} m_j n_{0j} \left[(1+\tau_j) \left(\frac{1-\nu w_j}{\sqrt{1-w_j^2}} - 1 \right) + \tau_j \ln\left(\frac{\nu}{\nu - w_j}\right) \right] \\ + \sum_j^{\gamma_j \neq 1} m_j n_{0j} \left\{ (1+\tau_j) \left(\frac{1-\nu w_j}{\sqrt{1-w_j^2}} - 1 \right) + \frac{\tau_j \gamma_j}{\gamma_j - 1} \left[\left(\frac{\nu}{\nu - w_j}\right)^{\gamma_j - 1} - 1 \right] \right\} = 0. \end{aligned} \tag{3.4}$$

This equation defines a curve in a space of normalized velocities w_j, which will be further referred to as a trajectory of the solution.

On the other hand, summing Eq. (3.1) over all species, using the Poisson equation, and integrating we have

$$\mathcal{B}(\mathbf{w}) \stackrel{\text{def}}{=} c^2 \sum_j m_j n_{0j} \left\{ \tau_j \left[\left(\frac{\nu}{\nu - w_j}\right)^{\gamma_j} - 1 \right] - \frac{\nu w_j (1+\tau_j)}{\sqrt{1-w_j^2}} \right\} = \frac{1}{8\pi} \left(\frac{d\phi}{d\xi}\right)^2. \tag{3.5}$$

Since the right hand side of Eq. (3.5) must be positive, configurations with velocities satisfying the inequality $\mathcal{B}(\mathbf{w}) < 0$ are not allowed. The boundary of this region is defined by

$$\sum_j m_j n_{0j} \left\{ \tau_j \left[\left(\frac{\nu}{\nu - w_j}\right)^{\gamma_j} - 1 \right] - \frac{\nu w_j (1+\tau_j)}{\sqrt{1-w_j^2}} \right\} = 0. \tag{3.6}$$

IV. TWO-COMPONENT PLASMA

Hereafter, the examination is restricted to a plasma consisting of two species. In this case, both the trajectory and the boundary are curves in the plane of the normalized velocities (w_1-w_2). The trajectory and the boundary curves can be represented as solutions of autonomous differential equations

$$\frac{dw_2^t}{dw_1} = \beta \frac{\left[(\nu - w_2)(1-w_2^2)^{3/2}\right] \left[\gamma_1 \tau_1 \left(\frac{\nu}{\nu - w_1}\right)^{\gamma_1 - 1} (1-w_1^2)^{3/2} - (1+\tau_1)(\nu - w_1)^2 \right]}{\left[(\nu - w_1)(1-w_1^2)^{3/2}\right] \left[\gamma_2 \tau_2 \left(\frac{\nu}{\nu - w_2}\right)^{\gamma_2 - 1} (1-w_2^2)^{3/2} - (1+\tau_2)(\nu - w_2)^2 \right]} \tag{4.1}$$

and

$$\frac{dw_2^b}{dw_1} = \beta \frac{\left[(\nu - w_2)(1-w_2^2)^{3/2}\right] \left[\gamma_1 \tau_1 \left(\frac{\nu}{\nu - w_1}\right)^{\gamma_1} (1-w_1^2)^{3/2} - (1+\tau_1)\nu(\nu - w_1) \right]}{\left[(\nu - w_1)(1-w_1^2)^{3/2}\right] \left[\gamma_2 \tau_2 \left(\frac{\nu}{\nu - w_2}\right)^{\gamma_2} (1-w_2^2)^{3/2} - (1+\tau_2)\nu(\nu - w_2) \right]}, \tag{4.2}$$

where $\beta = m_1 e_2/m_2 e_1$, with initial condition $w_2^t(0) = w_2^b(0) = 0$.

The initial point (0,0) belongs both to the trajectory and to the boundary, and at this point they have a common tangent

$$\left.\frac{dw_2^t}{dw_1}\right|_{w_2=w_1=w} = \left.\frac{dw_2^b}{dw_1}\right|_{w_2=w_1=w} = \beta\frac{\gamma_1\tau_1\left(\frac{\nu}{\nu-w}\right)^{\gamma_1-1}(1-w^2)^{3/2}-(1+\tau_1)(\nu-w)^2}{\gamma_2\tau_2\left(\frac{\nu}{\nu-w}\right)^{\gamma_2-1}(1-w^2)^{3/2}-(1+\tau_2)(\nu-w)^2}. \tag{4.3}$$

As one can see, for both the trajectory and the boundary, the extrema of w_2 achieve at $w_1 = \alpha_1$, where α_l is a root of the following equation:

$$\gamma_l\tau_l\left(\frac{\nu}{\nu-w_l}\right)^{\gamma_l-1}(1-w_l^2)^{3/2}-(1+\tau_l)(\nu-w_l)^2 = 0. \tag{4.4}$$

It is important to note that when $w_l < u < 1$ and $\gamma_l \geq 1$, this equation has only one root $-1 \leq w_l$. According to Eq. (2.2), the derivatives $dw_l/d\xi$ become infinite at $w_l = \alpha_l$, and therefore, the solution exists only on the part of the trajectory falling into the quadrant bounded by the lines $w_{1,2} = \alpha_{1,2}$ and containing the initial point (permissible quadrant).

Expressing $d\phi/d\xi$ from Eq. (3.5) and substituting it into Eq. (3.1) results in

$$m_2c^2\left[\frac{\gamma_2\tau_2(1-w_2^2)^{3/2}\left(\frac{\nu}{\nu-w_2}\right)^{\gamma_2-1}-(1+\tau_2)(\nu-w_2)^2}{(1-w_2^2)^{3/2}(\nu-w_2)}\right]\frac{dw_2}{d\xi}$$

$$= \pm e_2\sqrt{8\pi\sum_l m_lc^2 n_{0l}\left\{\tau_l\left[\left(\frac{\nu}{\nu-w_l}\right)^{\gamma_l}-1\right]-\frac{\nu w_l(1+\tau_l)}{\sqrt{1-w_l^2}}\right\}}. \tag{4.5}$$

Let $(\varkappa_1, \varkappa_2)$ be a point of the boundary and $\varkappa_l \neq \alpha_l$. Introducing new variables $y_l = w_l - \varkappa_l$, and taking into account that

$$y_2 = y_1\beta\frac{\left[(\nu-\varkappa_2)(1-\varkappa_2^2)^{3/2}\right]\left[\gamma_1\tau_1\left(\frac{\nu}{\nu-\varkappa_1}\right)^{\gamma_1-1}(1-\varkappa_1^2)^{3/2}-(1+\tau_1)(\nu-\varkappa_1)^2\right]}{\left[(\nu-\varkappa_1)(1-\varkappa_1^2)^{3/2}\right]\left[\gamma_2\tau_2\left(\frac{\nu}{\nu-\varkappa_2}\right)^{\gamma_2-1}(1-\varkappa_2^2)^{3/2}-(1+\tau_2)(\nu-\varkappa_2)^2\right]} + O(y_1^2), \tag{4.6}$$

due to Eq. (4.1), Eq. (4.5) takes the following form:

$$\sqrt{m_2c^2\frac{\gamma_2\tau_2\left(\frac{\nu}{\nu-\varkappa_2}\right)^{\gamma_2-1}(1-\varkappa_2^2)^{3/2}-(1+\tau_2)(\nu-\varkappa_2)^2}{(1-\varkappa_2^2)^{3/2}(\nu-\varkappa_2)}}\frac{dy_2}{d\xi} \tag{4.7}$$

$$= \pm e_2\sqrt{8\pi n_{0j}\nu y_2\frac{\varkappa_2-\varkappa_1}{(\nu-\varkappa_1)(\nu-\varkappa_2)}} + O(y_2^2). \tag{4.8}$$

For $\varkappa_1 = \varkappa_2$ (points where the bisector intersects the boundary), Eq. (4.7) can be reduced to

$$\frac{dy_2}{d\xi} = \pm a_2y_2 + O(y_2^2), \quad a_2 > 0, \tag{4.9}$$

and for $\varkappa_1 \neq \varkappa_2$

$$\frac{dy_2}{d\xi} = \pm b_2\sqrt{|y_2|} + O(|y_2|^{3/2}). \tag{4.10}$$

A solution of Eq. (4.9) behaves like an exponential function vanishing when $\xi \to \infty$. Hence, the trajectory may leave or enter points with $\varkappa_2 = \varkappa_1$ for infinite change of ξ. In contrast, Eq. (4.10) yields a finite change of ξ required to enter (leave) points where $\varkappa_2 \neq \varkappa_1$. Assuming $w_2 = w_1$ in Eq. (3.6), one can easily verify that the boundary and the bisector $w_2 = w_1$ have at most two common points. The infiniteness of the "escape time" for the point $(0, 0)$ proves the consistency of the boundary conditions (2.5).

At positive infinity the solution may have two different types of behavior. It can either end at the second point with the "infinite escape" time ($w_2 = w_1 \neq 0$) or reach a turning point on the boundary where $w_2 \neq w_1$ [at this point the sign of the right hand side of Eq. (4.5) changes] and return back to the initial point. In the first case the solution will have the form of a kink while the second one corresponds to a soliton. Obviously, the solution exists if and only if the following two conditions hold: (1) The trajectory touches the region $\mathcal{B} < 0$ at the initial point on the outer side (escape condition). (2) The trajectory intersects the boundary in the permissible quadrant.

The "escape condition" can be written in terms of second derivatives,

$$\left.\frac{d^2 w_2^b}{dw_1^2}\right|_{w_2=w_1=0} - \left.\frac{d^2 w_2^t}{dw_1^2}\right|_{w_2=w_1=0} = -\beta\frac{[\nu^2(1+\tau_2) - \beta\nu^2(1+\tau_1) - \gamma_2\tau_2 + \beta\gamma_1\tau_1][\gamma_1\tau_1 - \nu^2(1+\tau_1)]}{\nu[\gamma_2\tau_2 - (1+\tau_2)\nu^2]^2} =$$
$$= \frac{1}{\nu}\left.\frac{dw_2}{dw_1}\right|_{w_2=w_1=0}\left(1 - \left.\frac{dw_2}{dw_1}\right|_{w_2=w_1=0}\right). \tag{4.11}$$

If $\alpha_1 < 0$ and $\alpha_2 > 0$, the escape condition can be futher reduced yielding

$$0 < \left.\frac{dw_2}{dw_1}\right|_{w_2=w_1=0} = \beta\frac{\gamma_1\tau_1 - \nu^2(1+\tau_1)}{\gamma_2\tau_2 - \nu^2(1+\tau_2)} < 1. \tag{4.12}$$

If $\alpha_2, \alpha_1 > 0$ the escape condition can never be satisfied, while if $\alpha_2, \alpha_1 < 0$ it always holds.

Suppose that the escape condition is satisfied, i.e., in the neighborhood of the initial point the trajectory is in the region $\mathcal{B} > 0$. The requirement of intersection will then be fulfilled if the trajectory is in the region $\mathcal{B} < 0$ when it reaches one of the boundaries of the permissible quadrant. To show that this condition is also necessary, one has to prove that the trajectory and the boundary can have at most two common points in the permissible quadrant.

Let

$$f(w_1) \stackrel{\text{def}}{=} \frac{dg(w_1)}{dw_1} \stackrel{\text{def}}{=} \frac{d(w_2^t - w_2^b)}{dw_1}$$
$$= \beta\frac{\left[(\nu - w_2)^2(1-w_2^2)^{3/2}\right]\left[\gamma_1\tau_1\left(\frac{\nu}{\nu - w_1}\right)^{\gamma_1 - 1}(1-w_1^2)^{3/2} - (1+\tau_1)(\nu - w_1)^2\right]}{\left[(\nu - w_1)^2(1-w_1^2)^{3/2}\right]\left[\gamma_2\tau_2\left(\frac{\nu}{\nu - w_2}\right)^{\gamma_2 - 1}(1-w_2^2)^{3/2} - (1+\tau_2)(\nu - w_2)^2\right]}\left(\frac{w_2 - w_1}{\nu - w_2}\right), \tag{4.13}$$

taking into acount the definitions of α_k and β, it might be rewritten as

$$f(w_1) = \Gamma(w_1, w_2)\frac{(w_1 - \alpha_1)}{(w_2 - \alpha_2)}(w_1 - w_2), \tag{4.14}$$

where $\Gamma(w_1, w_2)$ is positive in the permissible quadrant. The function $f(w_1)$ is a single-valued function in the permissible quadrant. An assumption that the escape condition is satisfied yields

$$\mathrm{sgn} f(\delta) = -\mathrm{sgn}(\delta)\mathrm{sgn}(\alpha_2). \tag{4.15}$$

The first to show is that if $\alpha_1, \alpha_2 < 0$, the trajectory and the boundary have no points of intersection in the permissible quadrant ($w_1 > \alpha_1, w_2 > \alpha_2$). Let (w_1^*, w_2^*) be the point of intersection of the boundary and the trajectory immediately to the right of the initial point, $w_2^* < 0 < w_1^*$. Then Eq. (4.15) implies $f(+0) > 0$, i.e., $g(w_1)$ emerges from zero increasing and it has to decrease when it reaches its next zero, where $f(w_1^*) < 0$. According to Eq. (4.14), $w_2^* > w_1^*$. This, however, contradicts the assumption made before. The same argument can be applied if the point of intersection (w_1^*, w_2^*) is assumed to be immediately to the left of the initial point.

When $\alpha_2 < 0$, $\alpha_1 > 0$ the quadrant is defined by $w_1 < \alpha_1$, $w_2 > \alpha_2$. Again, let (w_1^*, w_2^*) be the point of intersection immediately to the right of the initial point, $w_1^* > 0$. Then Eq. (4.15) implies that $f(+0) > 0$, and $f(w_1^*) < 0$. However, this time Eq. (4.14) yields $w_2^* < w_1^*$. Note, that the escape condition implies $dw_2/dw_1 > 1$ and hence, the bisector $w_1 = w_2$ intersects the boundary between $w_1 = 0$ and $w_1 = w_1^*$. Suppose there exists another point of intersection (w_1^{**}, w_2^{**}) to the right of (w_1^*, w_2^*), $w_1^{**} > w_1^*$. Then, $g(w_1)$ reaches its zero at w_1^{**} increasing, i.e., $f(w_1^{**}) > 0$, and consequently $w_2^{**} > w_1^{**}$. It means that the line $w_1 = w_2$ should again intersect the boundary, which is impossible.

If (w_1^*, w_2^*) is the point of intersection immediately to the left, $w_1^* < 0$, then, since $f(-0) < 0$, we have $f(w_1^*) > 0$, and so $w_2^* > w_1^*$. This contradicts the condition $dw_2/dw_1 > 1$, which implies that the point is below the bisector, i.e., $w_2^* < w_1^*$. The case $\alpha_2 > 0, \alpha_1 < 0$ is obtained from the previous one by the formal change of indices.

As a result, the trajectory and the boundary, besides the initial point, may have only one other point of intersection in the permissible quadrant and only if $\alpha_l < 0$, $\alpha_k > 0$. Moreover, if (w_1^*, w_2^*) is such a point, then, $w_{1,2}^* > 0$. The sufficient condition of intersection which now becomes also necessary, can be written down as

$$w_1^t(\alpha_2) < w_1^b(\alpha_2). \tag{4.16}$$

The fact that $w_{1,2}^* > 0$ means that the velocities (and so the densities) can only increase, i.e., rarefactive solitons are not allowed.

V. EXISTENCE DOMAIN FOR COMPRESSIVE SOLITONS

As it was shown, the domain of existence for compressive solitons in a case $\alpha_2 > 0$, $\alpha_1 < 0$ is defined by (a) the escape condition, Eq. (4.12), and (b) the condition of intersection of the boundary and the trajectory in the permissible quadrant, Eq. (4.16).

Introducing new parameters $R_l = \tau_l(\gamma_l - \nu^2)/\nu^2$ and supposing that $\beta = -|\beta| = -m_1/m_2$, the escape condition takes the following form

$$0 < \left.\frac{dw_2}{dw_1}\right|_{w_2=w_1=0} = -|\beta|\frac{R_1-1}{R_2-1} < 1. \qquad (5.1)$$

The derivative is positive when $R_1 > 1$, $R_2 < 1$, and thus, temperatures of the corresponding species have to be $\tau_1 > \tau_1^{(cr)}$ and $\tau_2 < \tau_2^{(cr)}$, where $\tau_l^{cr} = \nu^2/(\gamma_l - \nu^2)$. The derivative is less than one, when $R_2 < 1 - |\beta|(R_1 - 1)$. Hence, the escape condition on a plane (R_1-R_2) is satisfied inside a triangle between lines $R_2 = 0$, $R_1 = 1$, and $R_2 = 1 - |\beta|(R_1 - 1)$. A line $R_2 = 1 - |\beta|(R_1 - 1)$ defines the right border of the domain and the equation

$$w_1^t(\alpha_2) = w_1^b(\alpha_2) \qquad (5.2)$$

defines its left border. Equation (5.2) is a compatibility condition of equations $\mathcal{T}(w_1, \alpha_2) = 0$ and $\mathcal{B}(w_1, \alpha_2) = 0$. Since they are transcendental equations, generally one has to solve them numerically. The compatibility condition is to be solved as a system of two nonlinear algebraic equations

$$\mathcal{T}(w_1, \tau_1, \alpha_2(\tau_2), \tau_2, \nu) = 0,$$
$$\mathcal{B}(w_1, \tau_1, \alpha_2(\tau_2), \tau_2, \nu) = 0. \qquad (5.3)$$

The unknown variables are w_1, τ_1, and the parameter τ_2 is to be changed from 0 to $\nu^2/(\gamma_2 - \nu^2)$ [when $\alpha_2(\tau_2) = 0$].

VI. ELECTRON-POSITRON PLASMA

In this section under electron-positron plasma, a plasma consisting of two species of equal masses is assumed.

In Figs. 1 and 2 the existence domains of compressive isothermal ($\gamma_e = \gamma_p = 1$) solitons are shown for $\nu = 0.01$ and $\nu = 0.9$, respectively. In the case of $\nu = 0.01$, $t_{e,p}^{cr} = 10^{-4}$, and in that of $\nu = 0.9$, $t_{e,p}^{cr} = 4.26$. For real electrons and positrons in the case $\nu = 0.9$, $T_{e,p}^{cr} = 2.66 \times 10^{10}$ K.

For a plasma consisting of two species, a heavy cold one i, and a light hot one e, in the nonrelativistic case

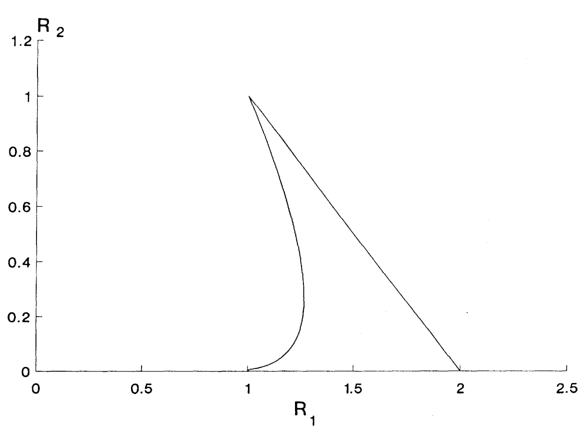

FIG. 1. The domain of existence of solitons for $\nu = 0.01$.

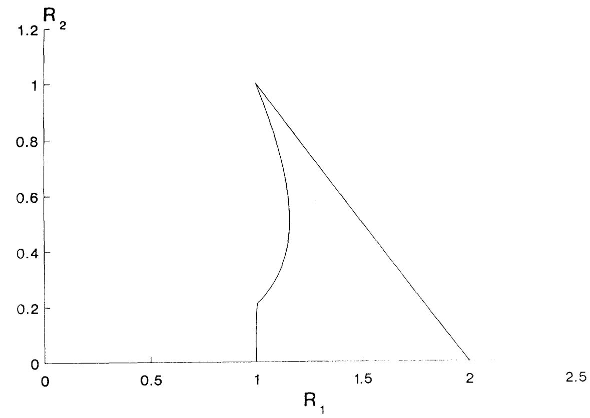

FIG. 2. The domain of existence of solitons for $\nu = 0.9$.

$R_e = m_i\gamma_e/\text{Ma}m_e$, where Ma= $u^2 m_i/T_e$ is the Mach number. In the relativistic case it is unclear what the Mach number is, since a relativistic acoustic wave does not exist. We, however, will use the Mach number here keeping in mind its restricted sense.

Obviously, the right border of the domain corresponds to the solitons with the lowest possible Mach numbers, whereas the left one corresponds to the highest ones ($R_e = 1$). In both the relativistic and the nonrelativistic cases, the bottom of the domain is defined by the escape condition. Hence, for a plasma with cold positrons, a range of possible Mach numbers can be easily obtained. It is the same for a whole variety of ν, $1/2 < \text{Ma} < 1$. As a result, in the sort of plasma under examination all solitons are "subsonic." As it appears from the figures, an area of the domain for the relativistic solitons, which is comprehensively defined by the escape condition, expands dramatically. The author does not have a rigorous mathematical proof yet, but the computation shows that the upper limit of this expansion is $R \approx 0.25$.

VII. DISCUSSION AND CONCLUDING REMARKS

The following results for a fluid model of a two-component plasma has been obtained: (a) The model does not admit rarefactive solitons, and (2) compressive solitons exist only in the domain defined by Eqs. (4.12) and (4.16).

It seems important to note that one does not have to do all these calculations in order to obtain a particular domain of existence. The important fact that the domain of existence of compresive solitons is comprehensively defined by the escape condition and the condition of intersection has been proved. These results coincide with those in the nonrelativistic case. This, in the author's opinion testifies to the promising potential of the method. Indeed, it is remarkable that in both the relativistic and the nonrelativistic cases, the trajectory and the boundary have the same properties: (1) They have a common tanget at the initial point, (2) the "escape time" is infinite only at common points, which are on the bisector, and (3) the boundary and the trajectory have at most two common points in the permissible quadrant. These properties allow us to clasify four kinds of solutions, compressive and rarefactive solitons, and kinks $[dn(\xi)/d\xi > 0]$ and antikinks $[dn(\xi)/d\xi < 0]$.

It is useful to note that these results remain unchanged in a case of a plasma with more than two species, if the additional species are the Boltzmann ones. Also, it is worth mentioning that Eq. (5.3) may be used to get the dependence of the solitons' amplitude on their speed. The unknown variables then are w_1 and w_2, and τ_1, τ_2, and ν are parameters chosen according to the domain of existence.

Obviously, it is much easier to solve a system of algebraic equations than the Sagdeev system. Although the method does not permit one to find an actual solution of the system of plasma fluid equations, a question arises: is it always of great importance to find an actual solution of the system, if using the method described here one can easily find nearly every important property of that solution? The author does not question importance and power of the Sagdeev pseudopotential method, but believes that in many cases it is not necessary to find an exact solution, especially given the fact that it is usually a numerical one.

The author failed to prove the absence of relativistic kinks as it was proven in [7], and has not considered solutions with trajectories intersecting the lines $w_l = \alpha_l$. Such solutions may be possible and would correspond to shock waves.

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